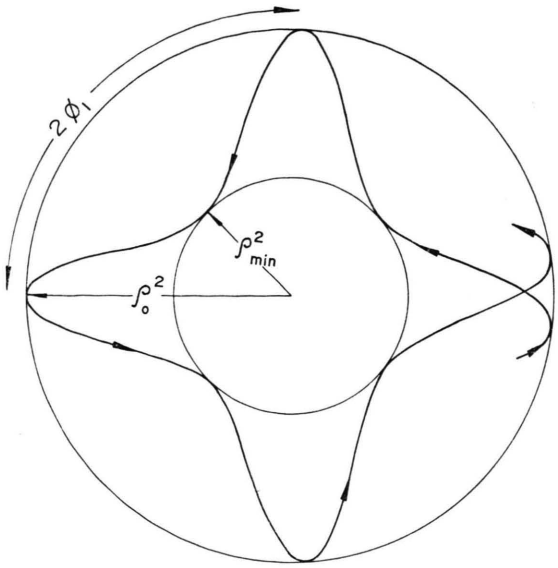

Fig. 2.

between the two turning points. Thus these projections are not closed curves. Figure 2 shows this typical curve.

Confinement of the Particle

The net u-displacement may be zero if the trajectory shows a loop in the $\varrho - z$ plane for $\beta < 1 - p_+$ and

$$(\omega^2 - \beta)\, F(\tfrac{1}{2}\,\pi, k) = \omega^2\, E(\tfrac{1}{2}\,\pi, k)\,. \tag{12}$$

This means that the particle moves between two coaxial cylinders and gets trapped between two transverse planes whose separation is

$$D = 2\,\{(\beta - \omega^2/\omega)\; F(\theta_1, k) + \omega\, E(\theta_1, k)\}\,. \tag{13}$$

The particle moves on a doughnut shaped surface whose section in the $\varrho - z$ plane is the curve C_5.

Curve C_6 shows that the particle may travel even against the plasma current. The flux invariance is obvious as the motion has z-periodicity.

I wish to express my thanks to Mr. N. D. SEN GUPTA for helpful discussions.

Nuclear Charge Radius Differences of $^{15}N - ^{14}N$ and $^{18}O - ^{16}O$

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The relative differences of the rms ground state nuclear charge radii of $^{15}N - ^{14}N$ and $^{18}O - ^{16}O$ have been measured by low energy elastic electron scattering to be $(1.3 \pm 0.7)\%$ and $(2.4 \pm 0.6)\%$, respectively. Both values are less than those following from an $A^{1/3}$ dependence.

The Darmstadt electron linear accelerator ($E \leqq 65$ MeV) has been used to measure the differences of the rms ground state nuclear charge radii R_m of $^{15}N - ^{14}N$ and $^{18}O - ^{16}O$ by elastic electron scattering at low momentum transfer ($q^2 < 0.25\ fm^{-2}$). Measurements at low momentum transfer have the advantage that radii and radius differences can be determined in a nearly model independent way [1, 2]. The experimental arrangement was that described by GUDDEN et al. [3], except for the 20-channel detector system in the focal plane of the spectrometer [4], the quadrupole doublet behind the scattering chamber [5], and a Faraday cage [6] instead of the old beam dump. Data were taken at a fixed incident electron energy of 50 MeV and scattering angles varying between 81° and 141° as well as at a fixed scattering angle of 93° and energies ranging from 30 to 65 MeV, giving a total of 13 data points.

The measurements were made on gaseous targets [7]. Cylindrical aluminum vessels whose walls were machined down to a thickness of about 200 μm were used as target cells. The gases were filled in through copper pipes which were pinched off after measuring the pressure. The pressure was measured by a precision quartz pressure gauge with a relative accuracy of better than 0.05%. It was choosen within the range from 0.4 to 0.8 atmospheres.

Three target cells were used in the experiment. Two of them contained either isotope, a third one was empty to determine the background. Effects due to differences ($< 5\%$) in the wall thicknesses of the two filled target cells were eliminated by repeating the measurement for each data point with the isotopes interchanged. The gas pressure in both cells was adjusted to the same value in order to avoid systematic errors. Thus, the local decrease in target gas density stemming from the heat generated by the beam along its path across the gas as well as dead time losses cancelled to a high

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degree in the cross section ratio. Corrections for differences in the relative detector efficiencies were avoided by changing the spectrometer magnetic field such that the elastic peak of either isotope fell into the same detector. In case of $^{18}O-{}^{16}O$, the whole measurement was done over again with two different target cells (and partly different energies and angles) in order to obtain an independent overall check of the entire procedure. The agreement between this and the previous measurement on $^{18}O-{}^{16}O$ was very good.

Corrections on the experimental cross sections for radiative and ionization effects cancelled to less than 0.3% in the cross section ratio. The data were analysed with a partial wave code by BÜHRING [8, 9]. The evaluated radius differences are not very sensitive to the model of charge distribution choosen in the analysis. For models compatible with the results of high energy electron scattering experiments on a number of p-shell nuclei, the model dependence has been estimated by BENTZ [10] to be less than 0.3% in R_m for our values of q^2. In our analysis, harmonic oscillator shell model type charge distributions [Eqs. (2) through (4) of [9]] were used. The fraction of magnetic dipole and electric quadrupole scattering from ^{15}N and ^{14}N was calculated in plane wave Born approximation to be less than 1% for $q^2 < 0.25\ \mathrm{fm}^{-2}$ and was taken into account.

The resulting relative radius differences

$$\Delta R_m/R_m = 2[R_m(2) - R_m(1)]/[R_m(2) + R_m(1)],$$

with (2) denoting the heavier, (1) the lighter isotope, are:

$$\Delta R_m/R_m(^{15}N-{}^{14}N) = (1.3 \pm 0.7)\%,$$
$$\Delta R_m/R_m(^{18}O-{}^{16}O) = (2.4 \pm 0.6)\%.$$

The errors include statistical as well as systematic effects. Systematic errors are mainly due to uncertainties in the background rate, in the collection losses of the Faraday cage, and in the radiative corrections. The model dependence is not included in the error.

Electron scattering experiments on ^{18}O and ^{16}O with the aim to measure differences of charge distribution parameters have been performed so far only at higher momentum transfer ($q^2 > 0.3\ \mathrm{fm}^{-2}$) [11, 12]. Assuming harmonic oscillator shell model type charge distributions, LACOSTE and BISHOP [11] found for the ratio of oscillator parameters $a(^{18}O)/a(^{16}O) = 1.025 \pm 0.006$. This value is equivalent to

$$\Delta R_m/R_m(^{18}O-{}^{16}O) = (2.5 \pm 0.6)\%.$$

SINGHAL et al. [12] measured

$$R_m(^{18}O)/R_m(^{16}O) = 1.020 \pm 0.005.$$

DANIEL et al. [13] studied muonic X-rays from ^{18}O and ^{16}O and obtained

$$\Delta R_m/R_m(^{18}O-{}^{16}O) = (3.5 \pm 0.8)\%.$$

All three values are in agreement with our result. Direct measurements of the $^{15}N-{}^{14}N$ radius differences have not been made so far. It should be noted that our measured $^{15}N-{}^{14}N$ and $^{18}O-{}^{16}O$ radius differences are smaller than what one would expect from an $A^{1/3}$ dependence (2.3% and 4.0%, respectively).

Combining our values of $\Delta R_m/R_m$ with the ^{14}N and ^{16}O radii obtained by BENTZ [10, 14], one deduces

$$R_m(^{15}N) = (2.52 \pm 0.04)\ \mathrm{fm}$$

and

$$R_m(^{18}O) = (2.73 \pm 0.04)\ \mathrm{fm}.$$

A more detailed account of this work will be published by one of us (W. S.) in „Zeitschrift für Physik".

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